Position and magnitude of balance mass required

Question:

Four masses m1, m2, m3 and m4 are 200 kg, 300 kg, 240 kg, and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of balance mass required, if its radius of rotation is 0.2 m.

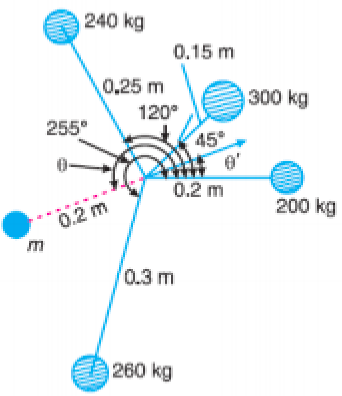
Answer:

Given : m_1 = 200 kg ; m_2 = 300 kg ; m_3 = 240 kg ; m_4 = 260 kg ; r_1 = 0.2 m ; r_2 = 0.15 m ; r_3 = 0.25 m ; r_4 = 0.3 m ; θ_1 = 0° ; θ_2 = 45° ; θ_3 = 45° + 75° = 120° ; θ_4 = 45° + 75° + 135° = 255° ; r = 0.2 m

Let m = Balancing mass, and

θ = The angle which the balancing mass makes with m_1.

Since the magnitude of centrifugal forces are proportional to the product of each mass and its radius, therefore



$$m_1 \cdot r_1 = 200 \times 0.2 = 40 \text{ kg-m}$$

$$m_2 \cdot r_2 = 300 \times 0.15 = 45 \text{ kg-m}$$

$$m_3 \cdot r_3 = 240 \times 0.25 = 60 \text{ kg-m}$$

$$m_4 \cdot r_4 = 260 \times 0.3 = 78 \text{ kg-m}$$

The problem may, now, be solved either analytically or graphically. But we shall solve the problem by both the methods one by one.

Analytical method

The space diagram is shown in Fig.

Resolving $m_1.r_1$, $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$ horizontally,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + m_3 \cdot r_3 \cos\theta_3 + m_4 \cdot r_4 \cos\theta_4$$

$$= 40 \cos 0° + 45 \cos 45° + 60 \cos 120° + 78 \cos 255°$$

$$= 40 + 31.8 - 30 - 20.2 = 21.6 \text{ kg-m}$$

Now resolving vertically,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + m_3 \cdot r_3 \sin\theta_3 + m_4 \cdot r_4 \sin\theta_4$$

$$= 40\sin 0° + 45\sin 45° + 60\sin 120° + 78\sin 255°$$

$$= 0 + 31.8 + 52 - 75.3 = 8.5 \text{ kg-m}$$

∴ Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(21.6)^2 + (8.5)^2} = 23.2$ kg-m

We know that $m \cdot r = R = 23.2$ or

Balancing mass $m = 23.2 / r = 23.2 / 0.2 = 116$ kg **Ans.**

and $\tan\theta' = \Sigma V / \Sigma H = 8.5 / 21.6 = 0.3935$ or $\theta' = 21.48°$

Since θ' is the angle of the resultant R from the horizontal mass of 200 kg, therefore the angle of the balancing mass from the horizontal mass of 200 kg,

$$\theta = 180° + 21.48° = 201.48°$$ **Ans.**